생체전자기학 연습문제

1. (20 점) Scalar field $T(x,y)=(x-3)^2+(y-1)^2$ 에 대해 답하시오.

   (a) $0 \le T \le 9$의 범위에서 contour plot 을 그리시오.
   (b) 점 (2, 0)에서 T와 ∇T를 계산하고 ∇T 를 contour plot 위에 그리시오.
   (c) 점 (2, 0)에서 ∇T의 크기와 방향의 의미를 기술하시오.
   (d) 점 (2, 0)에서 $(\nabla T \cdot \mathbf{a}_x)\mathbf{a}_x$와 $(\nabla T \cdot \mathbf{a}_y)\mathbf{a}_y$를 계산하고 contour plot 위에 그리시오.
   (e) 점 (0, 0)에서 $\nabla^2 T$를 계산하시오.

2. (20 점) 반경이 0.5 [mm]이고 길이가 50 [m]인 구리선이 원점 (0,0,0)에서 (0,0,50) [m] 사이에 놓여 있다. 원점을 기준 전위점으로 양단에 1 [V]의 전압을 인가했다. 구리의 도전율(conductivity)은 5.8×10^7 [S/m]이고, 자유전자의 밀도는 8.5×10^{28} [개/m^3]이며, 전자의 전하량은 -1.6×10^{-19} [C]이다.
   (a) 구리선 내부의 전기장 벡터 $\vec{\mathrm{E}}$를 구하시오.
   (b) 전류밀도 벡터 $\vec{\mathrm{J}}$를 구하시오.
   (c) 전체 전류 I를 구하시오.
   (d) 자유전자의 속력 벡터 $\vec{\mathrm{v}}$를 구하시오.
   (e) 구리선 양단의 전기저항 R을 구하시오.

3. (5 점) 점 A(0, 0, 1) [m]에 $4\pi\varepsilon_0$ [C]의 전하가 있고, 점 B(0, 0, –1) [m]에 $-4\pi\varepsilon_0$ [C]의 전하가 있다. 점 C(0, 0, 0)에서 전기장 벡터 $\vec{\mathrm{E}}$를 구하시오.

4. (5 점) 점 A(-1,0,0) [m]에 -1 [C]의 전하가 있고, 점 B(10,0,0) [m]에 1 [C]의 전하가 있다. 점 C(0,0,0)에서의 전위 V를 구하시오. 무한원점에서 전위가 0 [V]라 가정하시오.

5. (5 점) 2 차원 공간에 전기장 $\vec{\mathrm{E}}(x,y)=\vec{a}_x+x\vec{a}_y$ [V/m]이 존재한다. 점 A(1, 2)와 B(3, 5) 사이의 전위차 $V_\mathrm{B}-V_\mathrm{A}$를 구하시오.

6. Consider an artificial cell with a membrane that is permeable only to K^+ ion. Both intracellular and extracellular fluids are KCl solutions with different concentrations.
   (a) Derive the equation of the resting membrane potential V_{RMP} of the cell.
   (b) Assume that $\left[\mathrm{K}^+\right]_{in}=40\times\left[\mathrm{K}^+\right]_{out}$. Compute V_{RMP} and sketch the charge

distributions near the membrane.

(c) Sketch a block diagram of an instrument that measures V_{RMP}.

7. Assume a two-dimensional circular domain with a radius of 100 mm. It is filled with a saline of 1 S/m conductivity. A current dipole is placed at the center of the circle.
   (a) Formulate a boundary value problem for the potential $u(x, y)$ inside the circle.
   (b) Plot the current density streamlines.
   (c) Plot the equipotential lines.
   (d) Assume that you measure the voltage on the boundary of the circle at 40 points. Propose a method to detect the position and magnitude of the current dipole using the measured voltage data based on the lead field theory. You may discretize the circle with about 100 square elements. Assume that a current dipole can be located in each element.

8. The bioimpedance of the wrist changes with the pulsatile blood flow. You plan to measure the bioimpedance using 4 electrodes. One pair of the electrodes are for current injection and the other pair is for voltage measurement. Assume that the cross-section of the wrist is a circle with 50 mm diameter. Assume that the artery is located at the center of the wrist and its diameter changes from 2 mm to 4 mm.
   (a) Propose a best electrode configuration. Explain why it has a high sensitivity based on the lead field theory.
   (b) Propose a worst electrode configuration. Explain why it has a low sensitivity based on the lead field theory.

9. There is a cylindrical object filled with an electrolyte solution with conductivity σ. Its radius is r and height is $l = 5r$. A pair of electrodes with the surface area of πr^2 are attached at the top and bottom surfaces of the cylinder. The applied voltage at the bottom electrode is 10 V and the voltage at the top electrode is 0 V.
   (a) Formulate a boundary value problem for the potential u in the cylinder using the cylindrical coordinate system.
   (b) Solve the boundary value problem for the potential u and plot it.
   (c) Find the current density in the cylinder and plot it.
   (d) Find the total current flowing through the cylinder between the pair of electrodes.
   (e) Find the resistance between the pair of electrodes.
   (f) Assume that the size of both electrodes is reduced to $0.04\pi r^2$ and the electrodes are placed at the center of the top and bottom surfaces. Sketch the current density streamlines and equipotential lines inside the cylinder.
   (g) In the case of (f), discuss if there is any change (increase or decrease) in the

resistance value between the electrodes. Provide your explanation.

10. For mobile health applications, an engineer designed a battery-powered wrist band with three electrodes, which make good contacts with the skin around the wrist. Two of them are connected to the input terminals of a differential amplifier to measure ECG. The third electrode is connected to the floating circuit ground. Use the lead field theory to explain any technical problem(s) in this design. Suggest a better design to measure ECG.