Midterm Examination (Spring 2010)

(Answers should be either in English or Korean.)

(1) You are measuring a periodic blood pressure signal $p(t)$ from a patient with a heart rate of 60bpm (beat per minute). Since the signal happens to be an even function, you want to approximate it as $\hat{p}(t) = \sum_{n=0}^{5} A_n \cos(2\pi n t)$.

(a) Define an inner product space V for the function $p(t)$ with proper definitions of the inner product $\langle \cdot, \cdot \rangle$ and norm $\|\cdot\|$.

(b) Define a finite-dimensional subspace of V as W for the function $\hat{p}(t)$ and find its orthonormal basis.

(c) Find the orthogonal projection of $p(t)$ onto W. Compare $p(t)$, $\hat{p}(t)$, and error signal in terms of their energy, which is expressed as $\|\cdot\|^2$.

(d) You digitized the signal $p(t)$ using an ADC with 100Hz sampling frequency. Assuming that the error in (c) is negligible, propose an encoding and decoding method to transmit the signal.

(2) On a two-dimensional domain $\Omega = \{(x,y) \,|\, |x| \le 10\text{m} \text{ and } |y| \le 10\text{m}\}$, you measured voltage $v(x,y)$ and found that $v(x,y) = x^2 + y^2$ in Ω.

(a) Draw a contour plot of $v(x,y)$ in Ω including at least 5 contours.

(b) Compute and draw $\nabla v(x,y) / |\nabla v(x,y)|$ at three points of (0,5m), (5m,0), and (5m,5m).

(c) Assume that you are trying to find the position where the voltage is minimal without knowing the voltage v in Ω. You made the first voltage measurement at (5m,5m). You are going to make the second voltage measurement at a point which is 2m away from the first point. Decide the best second point. Draw the first and second measurement points on the contour plot in (a).

(d) Repeat (a), (b) and (c) for $v(x,y) = x^2 + \frac{y^2}{25}$.

(3) Assume a two-dimensional circular bucket with a radius of 10cm. It is filled with a saline of 1S/m conductivity. You dropped a battery in the bucket. Positions of its positive and negative terminals happen to be (0,1cm) and (0,-1cm), respectively.

(a) Formulate a boundary value problem for the voltage $v(x,y)$ inside the

bucket. You may express the source or sink as $f(x,y)$.

(b) Plot $f(x,y)$ and streamlines of current density distribution.

(c) Plot equipotential lines (contour plot of *v*).

(d) Using a voltmeter with its reference probe at (0,0), you measured voltage inside the bucket at all grid points with $\Delta_x=\Delta_y=0.1$cm. Propose an algorithm to detect source and sink.

(4) Solve the following boundary value problem using the finite element method. Your mesh includes 2 linear elements with the same size.

$$\frac{d}{dx}\left(\frac{du(x)}{dx}\right)=0,\ 0\le x\le 10,\ u(0)=10,\ u(10)=0$$

(5) You are developing a digital camera where color is expressed as three 8-bit numbers of (*R*,*G*,*B*). You found that measured color is distorted in all three components. You took several pictures of standard color sheets and obtained the following data. You want to improve the image quality in terms of a sense of color.

True Color			Measured Color		
R	*G*	*B*	R_m	G_m	B_m
0	0	0	3	2	1
50	0	0	60	0	0
0	50	0	0	48	0
0	0	50	0	0	40
100	0	0	135	0	0
0	100	0	0	103	0
0	0	100	0	0	72
150	0	0	190	0	0
0	150	0	0	155	0
0	0	150	0	0	110
200	0	0	210	0	0
0	200	0	0	198	0
0	0	200	0	0	187

(a) Plot R_m, G_m, and B_m as functions of *R*, *G*, and *B*, respectively.

(b) Set $R_m=a_1R^2+b_1R+c_1$, $G_m=a_2G^2+b_2G+c_2$, and $B_m=a_3B^2+b_3B+c_3$. Using the least square method, determine $\mathbf{p}=[a_1\ b_1\ c_1\ a_2\ b_2\ c_2\ a_3\ b_3\ c_3]^T$.

(c) Explain how to compensate color distortions.