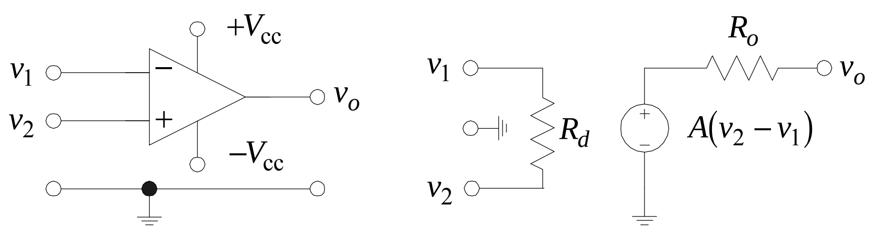
Chapter 3. Amplifiers and Signal Processing

3.1 Ideal OP Amps

- Op amp
  - High-gain dc differential amplifier
  - Dc power supplies are required
  - Usually used with external negative feedback
- Assume ideal op amp ⇒ design circuit ⇒ check nonideal characteristics are important ⇒ modify if necessary



Ideal Characteristics

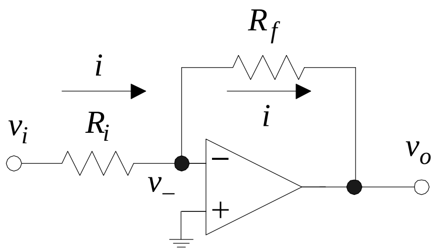
- $A = \infty$ (gain is infinite)
- $v_o = 0$ when $v_1 = v_2$ (no offset voltage)
- $R_d = \infty$ (input impedance is infinite)
- $R_o = 0$ (output impedance is zero)
- Bandwidth $= \infty$ (no frequency response limitation) and no phase shift

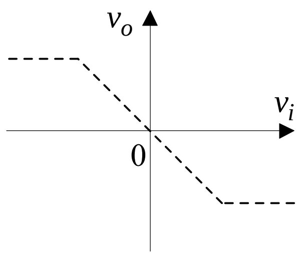
Two Basic Rules

- *Rule 1* When the op amp output is in its linear region, the two input terminals are at the same voltage.
- *Rule 2* No current flows into either input terminal of the op amp.
- Saturation of output at slightly lower than power supply voltages

3.2 Inverting Amplifiers

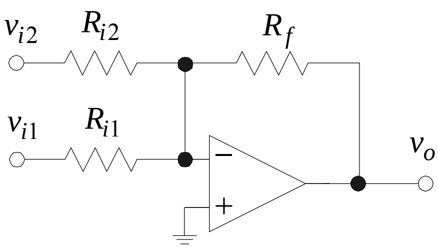
- Inverting amplifier and input-output characteristic





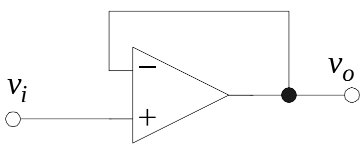
- Virtual ground: $v_- = 0$
- Analysis: $v_o = -i\,R_f = -v_i \dfrac{R_f}{R_i}$ or $\dfrac{v_o}{v_i} = -\dfrac{R_f}{R_i}$
- $R_{in} = R_i$ and $R_{out} = 0$

❍ Summing amplifier: $v_o = -R_f \left(\dfrac{v_{i1}}{R_{i1}} + \dfrac{v_{i2}}{R_{i2}} \right)$



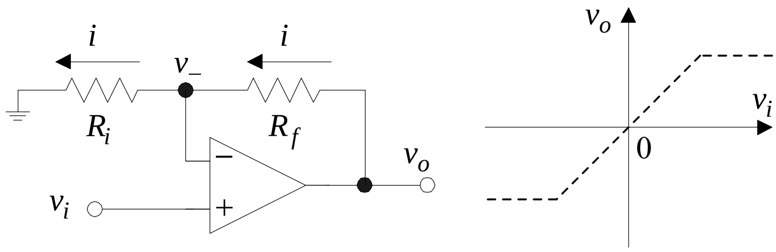
3.3 Noninverting Amplifiers

❍ Unity-gain follower or buffer: $v_o = v_i$, $R_{in} = \infty$, and $R_{out} = 0$

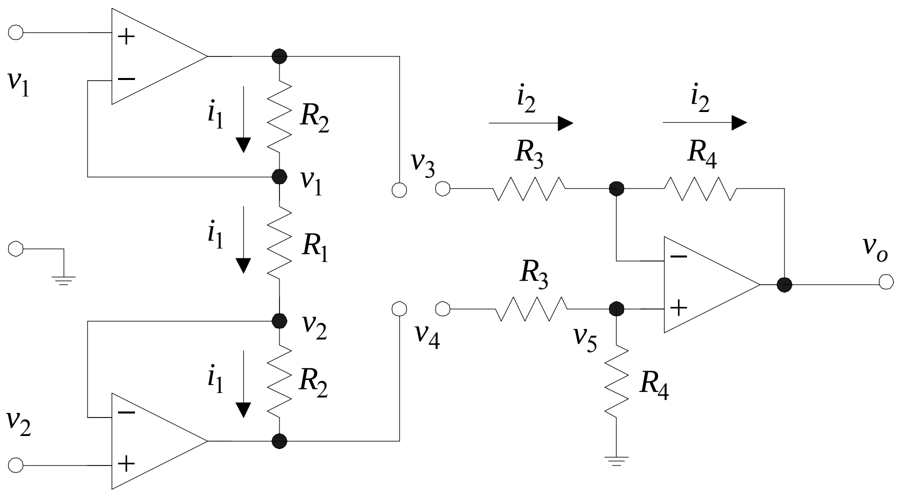


❍ Noninverting amplifier and input-output characteristic

- $v_- = v_i$
- Analysis: $\dfrac{v_o}{v_i} = \dfrac{i\left(R_f + R_i\right)}{i\,R_i} = 1 + \dfrac{R_f}{R_i}$
- $R_{in} = \infty$ and $R_{out} = 0$



3.4 Differential Amplifiers



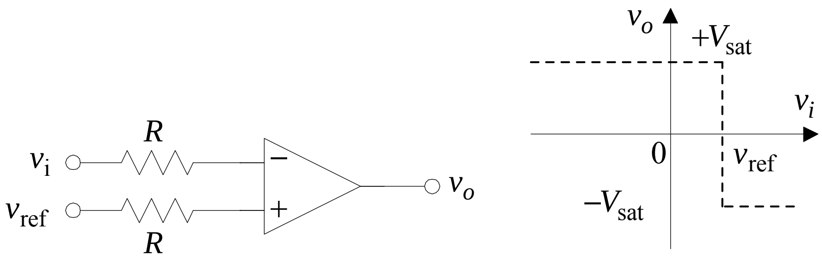
- One-op-amp differential amplifier
  - Analysis: $v_5 = \dfrac{R_4}{R_3 + R_4} v_4$, $i_2 = \dfrac{v_3 - v_5}{R_3} = \dfrac{v_5 - v_o}{R_4}$, $v_o = \dfrac{R_4}{R_3}(v_4 - v_3)$
  - Common-mode rejection ratio (*CMRR*): $CMRR = \dfrac{G_d}{G_c}$ or $20\log\dfrac{G_d}{G_c}$ dB
    - Differential mode gain, $G_d = \dfrac{R_4}{R_3}$
    - Common model gain, G_c: gain for $v_3 = v_4$
  - $R_{in} < \infty$ (could be small) and $R_{out} = 0$
- Three-op-amp differential amplifier (instrumentation amplifier)
  - Analysis: $v_3 - v_4 = i_1(2R_2 + R_1)$, $v_1 - v_2 = i_1 R_1$, $v_3 - v_4 = \left(1 + 2\dfrac{R_2}{R_1}\right)(v_1 - v_2)$,

$$v_o = \left(1 + 2\frac{R_2}{R_1}\right)\frac{R_4}{R_3}(v_2 - v_1)$$

- Common-mode rejection ratio ($CMRR$): $CMRR = \frac{G_d}{G_c}$ or $20\log\frac{G_d}{G_c}$ dB
    - Differential mode gain, $G_d = \left(1 + 2\frac{R_2}{R_1}\right)\frac{R_4}{R_3}$
    - Common model gain, G_c : gain for $v_1 = v_2$
- $R_{in} = \infty$ and $R_{out} = 0$

3.5 Comparators

❍ Simple comparator or Schmitt trigger



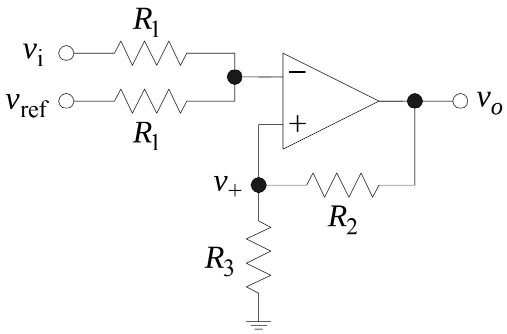
- $v_i > v_{ref} \Rightarrow v_o = -V_{sat}$ and $v_i < v_{ref} \Rightarrow v_o = +V_{sat}$
- R minimizes overdriving op amp input.
- Sensitive to noise at input
- v_i and v_{ref} can be interchanged.

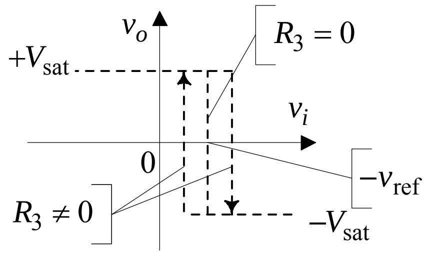
❍ Comparator with hysteresis

- Positive feedback with R_2 and $R_3 \Rightarrow$ hysteresis $\Rightarrow$ insensitive to input noise
- Analysis
    - $v_o = +V_{sat} \Rightarrow v_+ = \frac{R_3}{R_2 + R_3}V_{sat} \Rightarrow v_i$ must be greater than $-v_{ref} + \frac{R_3}{R_2 + R_3}V_{sat}$ to produce $v_o = -V_{sat}$

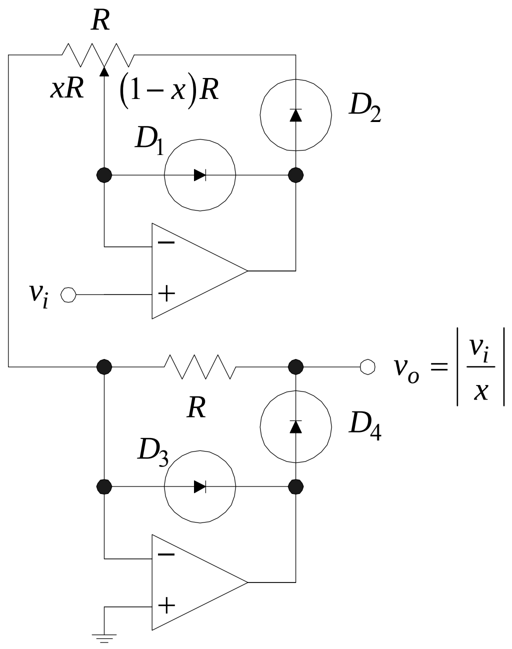
- $v_o = -V_{sat} \Rightarrow v_+ = -\frac{R_3}{R_2+R_3}V_{sat} \Rightarrow v_i$ must be smaller than $-v_{ref} - \frac{R_3}{R_2+R_3}V_{sat}$ to produce $v_o = +V_{sat}$

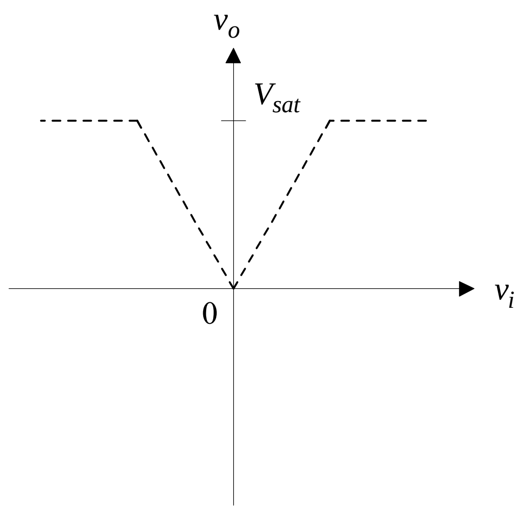
- R_3 controls the width of the hysteresis.





3.6 Rectifiers



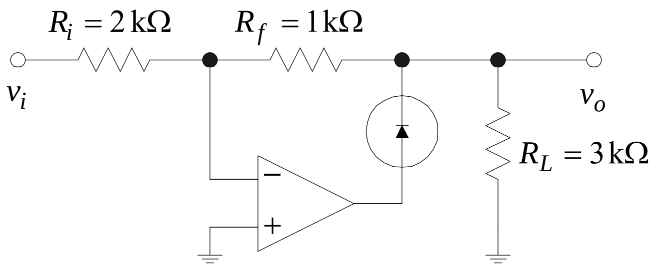


- Full-wave precision rectifier
  - $v_i > 0 \Rightarrow D_2$ and D_3 ON, D_1 and D_4 OFF $\Rightarrow$ upper op amp circuit becomes a noninverting amplifier with gain of $1/x$, lower op amp circuit has no effect on output

- $v_i < 0 \Rightarrow D_2$ and D_3 OFF, D_1 and D_4 ON $\Rightarrow$ upper op amp circuit has no effect on output becomes a noninverting amplifier with gain of $1/x$, lower op amp circuit becomes an inverting amplifier with gain of $-1/x$
- Variable gain and high input impedance

❍ Half-wave precision rectifier

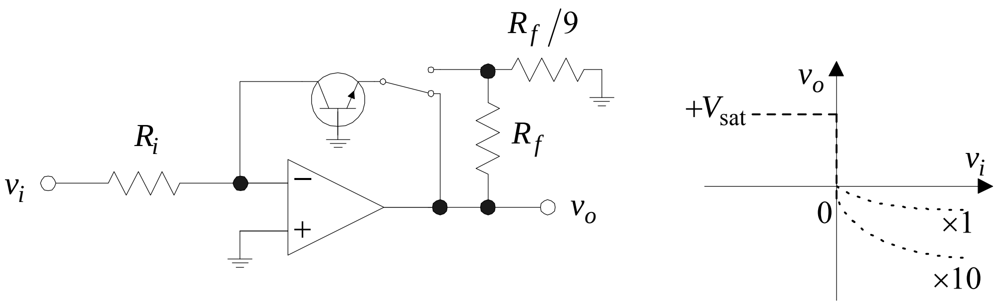
- Upper or lower op amp circuit



❍ One op-amp full-wave rectifier

- Gain is a function of load $\Rightarrow$ constant load is required

3.7 Logarithmic Amplifiers



❍ Without boosting

- For a transistor, $V_{BE} = 0.060\log\left(\frac{I_C}{I_S}\right)$ with reverse saturation current

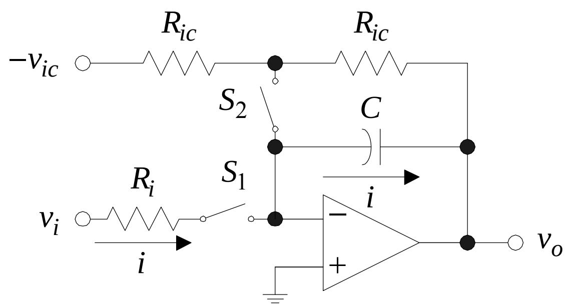
  $I_S = 10^{-13}\,\text{A}$ at 27 °C

- Transdiode configuration: $I_C = \frac{v_i}{R_i}$ and $v_o = V_{BE} = 0.060\log\left(\frac{v_i}{I_S R_i}\right)$

- For $10^{-7}\ \text{A} < I_C < 10^{-2}\ \text{A}$, $-0.66\ \text{V} < v_o < -0.36\ \text{V}$

- With boosting: same as noninverting amplifier
- Temperature compensation for accuracy
- Antilog (exponential) circuit: interchange resistor with transistor
- Applications
  - Multiplication, division, power
  - Dynamic range compression
  - Linealization

3.8 Integrators



- Integrator
  - Initial condition setting: S_1 open and S_2 closed $\Rightarrow v_o = v_{ic}$ (inverting amplifier) and $v_c(0) = -v_{ic}$
  - Integration: S_1 closed and S_2 open $\Rightarrow v_c = \frac{1}{C}\int_0^{t_1} i\,dt - v_{ic}$ and $i = \frac{v_i}{R}$ $\Rightarrow$
    $v_o = -\frac{1}{RC}\int_0^{t_1} v_i\,dt + v_{ic}$
  - Hold: S_1 open and S_2 open $\Rightarrow v_o$ is hold
  - Frequency response: $\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega RC} = -\frac{1}{j\omega\tau}$
  - Drift and saturation problem

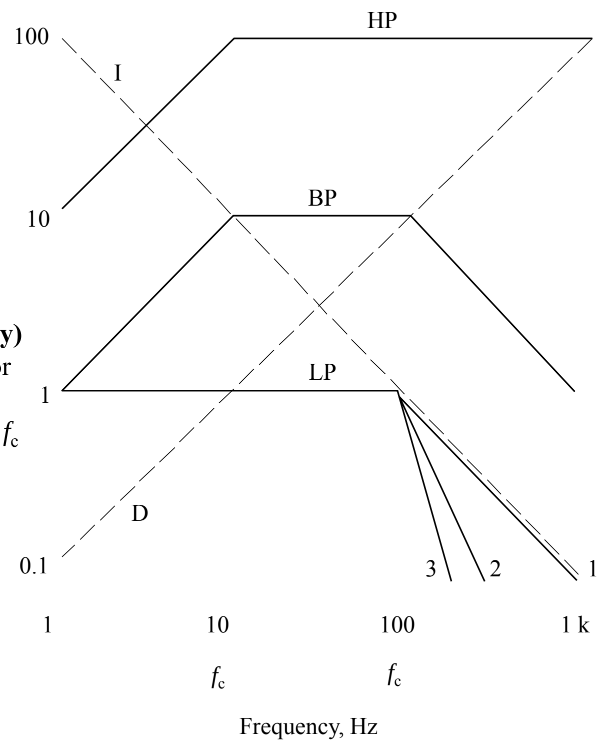
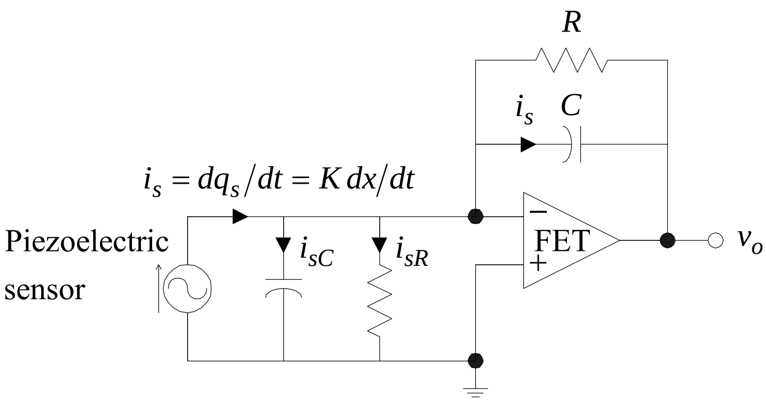
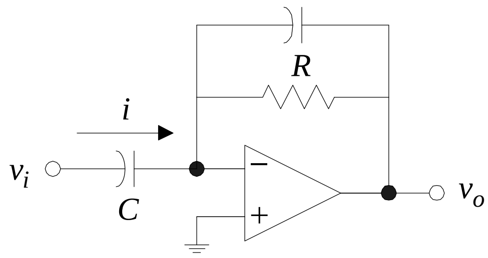


Figure 3.10 Bode plot (gain versus frequency) for various filters. Integrator (I); differentiator (D); low pass (LP), 1, 2, 3 section (pole); high pass (HP);bandpass (BP). Corner frequencies f_c for high-pass, low-pass, and bandpass filters.

- Charge amplifier
  - Virtual ground $\Rightarrow$ $i_{sC} = i_{sR} = 0$ $\Rightarrow$ long cable can be used
  - $i_s = K\dfrac{dx}{dt} \Rightarrow v_o = -\dfrac{1}{C}\displaystyle\int_0^{t_1} K\dfrac{dx}{dt}dt = -\dfrac{Kx}{C}$
  - Drift and saturation problem
  - Large feedback resistor
    - Prevents saturation
    - Highpass filter with $f_c = \dfrac{1}{2\pi RC}$ $\Rightarrow$ no frequency response improvement over voltage amplifier



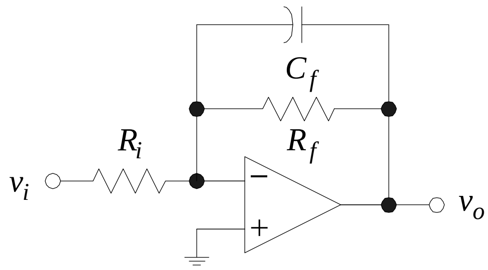
3.9 Differentiators



- Differentiator
  - $i = C\dfrac{dv_i}{dt}$ and $v_o = -Ri = -RC\dfrac{dv_i}{dt}$
  - Frequency response: $\dfrac{V_o(j\omega)}{V_i(j\omega)} = -\dfrac{Z_f}{Z_i} = -\dfrac{R}{1/j\omega C} = -j\omega RC = -j\omega\tau$
- Differentiator output: tends to oscillate and noisy due to amplification of high frequency components

3.10 Active Filters

Low-Pass Filter



- Frequency response:

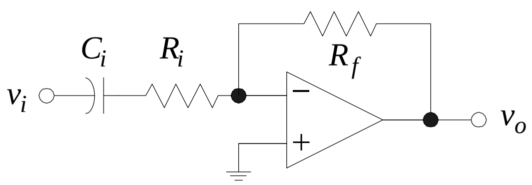
$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{\dfrac{R_f / j\omega C_f}{\left[\left(1/j\omega C_f\right) + R_f\right]}}{R_i} = -\frac{R_f}{\left(1 + j\omega R_f C_f\right) R_i} = -\frac{R_f}{R_i}\frac{1}{\left(1 + j\omega\tau\right)}$$

▫ If $\omega << 1/\tau$ or $f << f_c$ with $f_c = 1/2\pi R_f C_f$ circuit becomes inverting amplifier with gain $-R_f / R_i$

▫ If $\omega >> 1/\tau$ or $f >> f_c$, circuit becomes integrator

- Cutoff frequency or corner frequency: $f_c = 1/2\pi R_f C_f$

High-Pass Filter



- Frequency response:

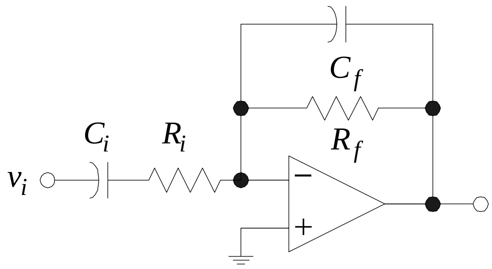
$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R_f}{\left(1/j\omega C_i\right) + R_i} = -\frac{j\omega R_f C_i}{1 + j\omega R_i C_i} = -\frac{R_f}{R_i}\frac{j\omega\tau}{1 + j\omega\tau}$$

▫ If $\omega << 1/\tau$ or $f << f_c$ with $f_c = 1/2\pi R_i C_i$, circuit becomes differentiator

▫ If $\omega >> 1/\tau$ or $f >> f_c$, circuit becomes inverting amplifier with gain $-R_f / R_i$

- Cutoff frequency or corner frequency: $f_c = 1/2\pi R_i C_i$

Band-Pass Filter



- Series combination of lowpass and highpass filter
- Two cutoff frequencies or corner frequencies: $f_{c1} = 1/2\pi R_i C_i$ and $f_{c2} = 1/2\pi R_f C_f$

  with $f_{c2} > f_{c1}$
  - If $f << f_{c1}$, circuit becomes differentiator
  - If $f >> f_{c2}$, circuit becomes integrator

3.11 Frequency Response

❍ For a real op amp, bandwidth is not infinite.

Open-Loop Gain

- Op amp is multi-stage dc differential amplifier with high gain
- Stray or junction capacitances in each stage ⇒ gain attenuation (–1 slope on log-log plot and –90° phase shift per stage) ⇒ slope changes with frequency
- Real op amp has a limited open-loop bandwidth
- Possible oscillation (gain greater than 1 at –180° phase shift)

Compensation

- Add a capacitor (external or internal) ⇒ fixed slope of –1 and maximal phase shift of –90°, open-loop cutoff frequency of about 40 Hz
- No oscillation

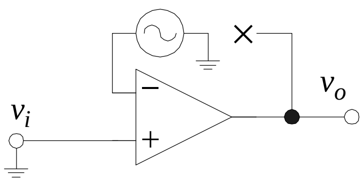
Closed-Loop Gain

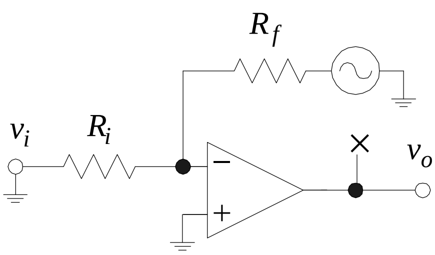
- Closed-loop gain is usually much smaller than the maximal open-loop gain of op amp.
- Closed-loop gain is determined by external elements forming negative feedback.

- Closed-loop gain can never exceed open-loop gain.

Loop Gain

- Loop gain = (open-loop gain of op amp) – (closed-loop gain of op amp circuit)
    - At low frequency: high loop gain, external feedback circuit determines the op amp circuit
    - At high frequency: low loop gain, the op amp circuit follows the op amp open-loop gain
    - High loop gain ⇒ high accuracy and stability
- Measurement of loop gain
    - Break feedback loop at any point in the loop
    - Inject a signal
    - Measure the gain around the loop
    - Examples
        - Unity gain follower: loop gain = open-loop gain





- Inverting amplifier with gain of –1: loop gain = (open-loop gain)/2

Gain-Bandwidth Product

- Gain-bandwidth product = (gain at *f*) × (bandwidth at *f*)
- Unity-gain-bandwidth product is given in specification of op amp
- Compensated op amp has gain slope of –1 ⇒

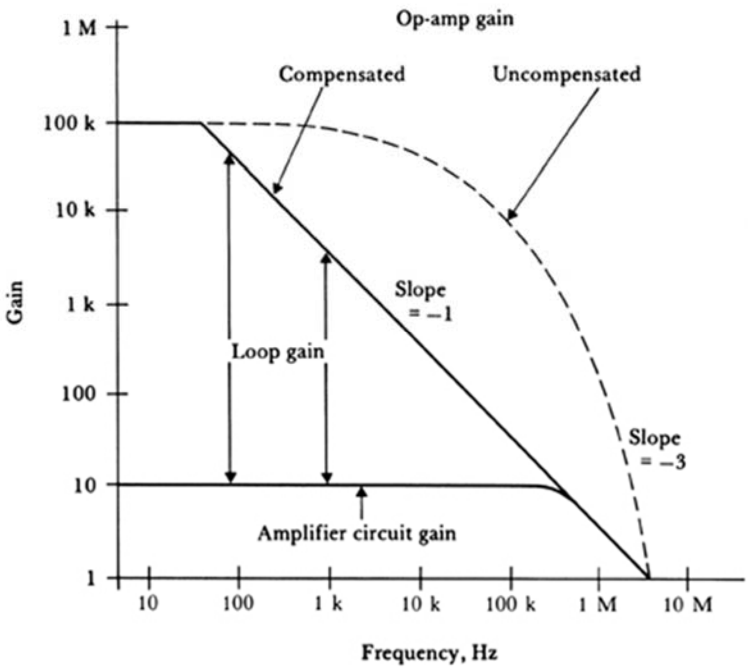
$$\text{Bandwidth of op amp circuit} = \frac{\text{Unity - gain - bandwidth product in Hz}}{\text{Op amp circuit gain}}$$

Slew Rate

- For an op amp, internal current source has its $I_{\max}$.
- Change in voltage across the compensation capacitor: $\frac{dv_c}{dt} = \frac{I_{\max}}{C} \Rightarrow \frac{dv_o}{dt}$ is limited

- Slew rate $S_r = \left.\dfrac{dv_o}{dt}\right|_{\max}$
- For sinusoidal input, full-power response or maximal frequency for rated output is $f_p = \dfrac{S_r}{2\pi V_{or}}$ where V_{or} is the rated output voltage.
- Uncompensated op amp is faster $\Rightarrow$ useful for comparators

Figure 3.13 Op-amp frequency characteristics early op amps (such as the 709) were uncompensated, had a gain greater than 1 when the phase shift was equal to –180°, and therefore oscillated unless compensation was added externally. A popular op amp, the 411, is compensated internally, so for a gain greater than 1, the phase shift is limited to –90°. When feedback resistors are added to build an amplifier circuit, the loop gain on this log-log plot is the difference between the op-amp gain and the amplifier-circuit gain.



3.12 Offset Voltage

- For a real op amp, $v_2 - v_1 \neq 0$ to produce $v_o = 0$.
- Offset voltage = $v_2 - v_1 \neq 0$ must be considered for small input signals.

Nulling

- Add an external nulling pot.
- Adjust the pot $\Rightarrow$ increase I_E at one input and decrease at the other $\Rightarrow$ $v_2 - v_1 = 0$

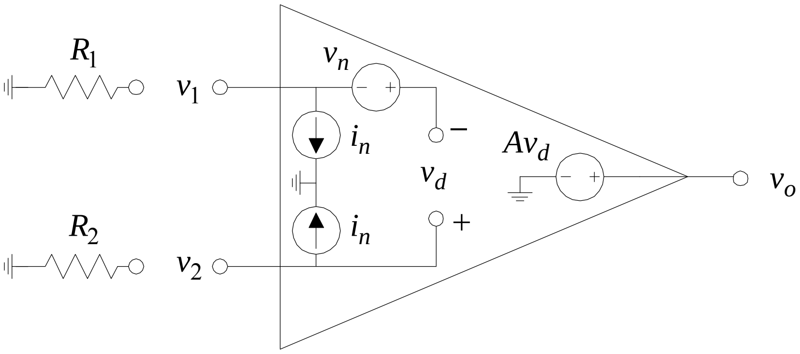
Drift

- Temperature change (environment or self-heating) $\Rightarrow$ change in offset voltage, $(v_2 - v_1)$

- Specification
  - Maximal offset voltage change per °C such as 0.1 μV/°C
  - Maximal offset voltage over a given temperature range such as –25 to +85 °C

Noise

- Semiconductor junctions ⇒ noise voltage sources and noise current sources
- For low source impedance (R_1 and R_2 small), v_n dominates.
- Characteristics
  - Random
  - At low frequency ⇒ amplitude $\propto 1/f$ (flicker noise)
  - At midfrequency ⇒ smaller amplitude expressed in rms units of $\mathrm{V}\cdot\mathrm{Hz}^{-1/2}$
  - Some op amps exhibit bursts of noise (popcorn noise).

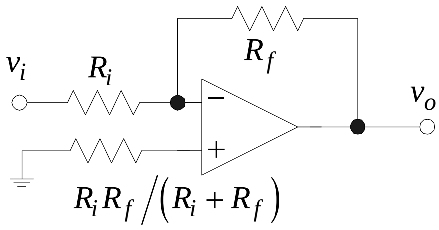


3.13 Bias Current

- Base or gate current to keep transistors turned on ⇒ bias current ≠ 0
- Bias current flows through feedback resistors ⇒ smaller resistors are desirable (about 10 kΩ)
- Caution: current flowing through feedback resistors plus current flowing through loads must be smaller than op amp output current rating ⇒ too small resistors cannot be used

Differential Bias Current

- Difference between two input bias currents << each bias current
- Compensation resistor minimizes the effect of bias currents



Drift

- Change of bias currents due to temperature
- Compensation resistor also minimizes the effect of bias current drift

Noise

- Noise currents flow through external equivalent resistors.

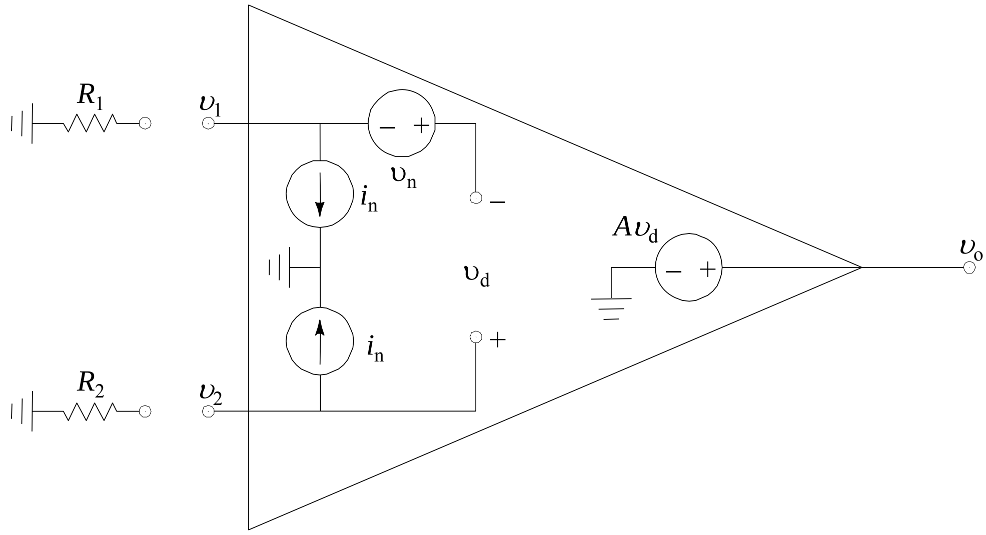
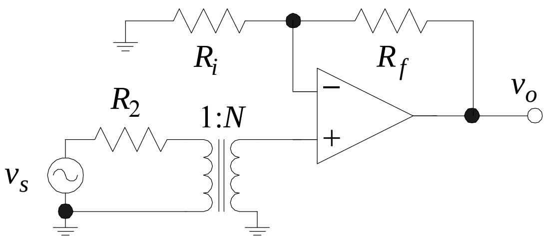


Figure 3.14 Noise sources in an op amp The noise-voltage source v_n is in series with the input and cannot be reduced. The noise added by the noise-current sources In can be minimized by using small external resistances.

- Total rms noise voltage is $v_t \cong \left[\left\{ v_n^2 + (i_n R_1)^2 + (i_n R_2)^2 + 4kTR_1 + 4kTR_2 \right\} \times BW \right]^{1/2}$
  - R_1 and R_2 : equivalent source resistances
  - v_n : mean value of rms noise voltage in $\mathrm{V \cdot Hz^{-1/2}}$ over a frequency range
  - i_n : mean value of rms noise voltage in $\mathrm{A \cdot Hz^{-1/2}}$ over a frequency range

- k: Boltzmann's constant
- T: temperature, K
- BW: noise bandwidth, Hz

- Types of op amp
  - Small (10 kΩ) source resistances ⇒ BJT input op amp produces smaller noise
  - Large source resistances ⇒ FET input op amp produces smaller noise due to smaller noise current
- Low noise ac amplifier design (noninverting amplifier) by impedance matching
  - Characteristic noise resistance is $R_n = v_n / i_n$
  - Set $R_n = R_2$ using a transformer with turns ratio 1:N where $N = (R_n/R_2)^{1/2}$



3.14 Input and Output Resistance

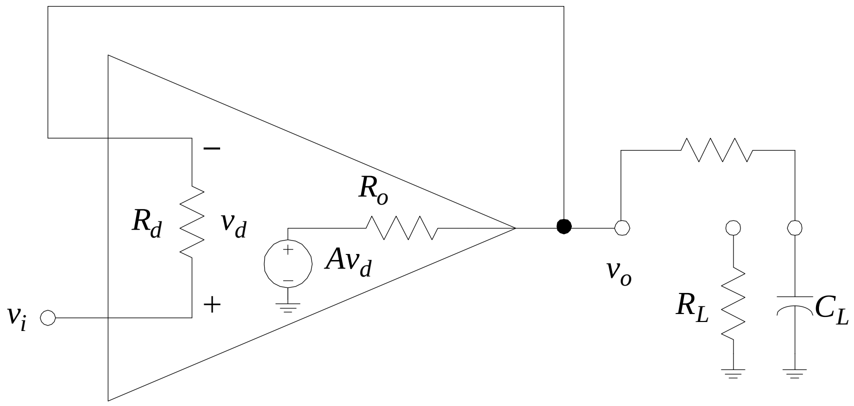
Input Resistance

- Op amp differential input resistance, R_d : TΩ for FET, MΩ for BJT
- Amplifier-circuit input resistance, R_{ai}
  - Unity-gain follower: $\Delta v_o = A\Delta v_d = A(\Delta v_i - \Delta v_o) \Rightarrow \Delta v_o = \frac{A\Delta v_i}{A+1}$

$$\Delta i_i = \frac{\Delta v_d}{R_d} = \frac{\Delta v_i - \Delta v_o}{R_d} = \frac{\Delta v_i}{(A+1)R_d}$$

$$R_{ai} = \frac{\Delta v_i}{\Delta i_i} = (A+1)R_d \approx AR_d \text{ , } R_{ai} \text{ could be} > \text{T}\Omega$$

  - Noninverting amplifiers: $R_{ai} = R_d \times (\text{loop gain})$, very high, limited by surface leakage current
  - Inverting amplifier: $R_{ai} = \frac{\Delta v_i}{\Delta i_i} = R_i$, usually small



Output Resistance

- Op amp output resistance, $R_o \approx 40\ \Omega$
- Amplifier-circuit output resistance, R_{ao} for unity-gain follower with resistive load
  - Resistive load, $R_L \Rightarrow$ change in output current, Δi_o

$$-\Delta v_d = \Delta v_o = A\Delta v_d + \Delta i_o R_o = -A\Delta v_o + \Delta i_o R_o \Rightarrow (A+1)\Delta v_o = \Delta i_o R_o$$

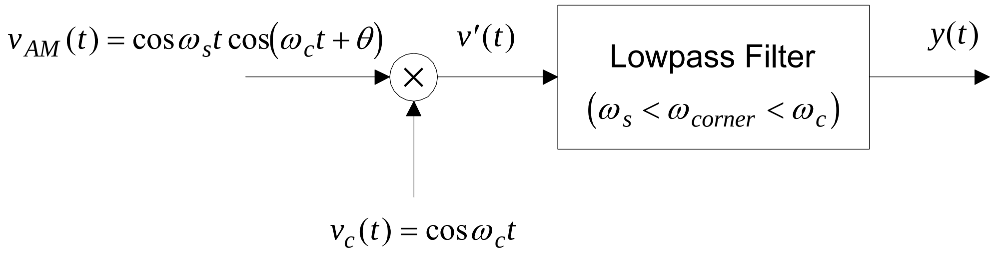
$$R_{ao} = \frac{\Delta v_o}{\Delta i_o} = \frac{R_o}{A+1} \approx \frac{R_o}{A},\ R_{ao} \text{ could be} < 10\text{-}3\ \Omega$$

  - All noninverting and inverting amplifiers: $R_{ao} = R_d/(\text{loop gain})$, very small, load resistance is limited by maximal output current of op amp (too small load resistance $\Rightarrow$ op amp saturates internally)
- R_{ao} for unity-gain follower with capacitive load
  - Capacitive load, $C_L \Rightarrow i_o = C_L \dfrac{dv_o}{dt}$, limited by maximal output current of op amp and slew rate
  - $R_o - C_L \Rightarrow$ lowpass filter $\Rightarrow$ additional phase shift around the loop $\Rightarrow$ possible oscillation
  - To prevent oscillation, add a small resistor between v_o and C_L.
- Current booster for large output current: op amp + high-power transistors

3.15 Phase-Sensitive Demodulators

- Consider the amplitude-modulated (AM) signal, $v_{AM}(t) = x(t)\cos\omega_c t$
  - Signal: $x(t)$ with maximal frequency much less than $f_c = \omega_c/2\pi$
  - Carrier: $\cos\omega_c t$ with $\omega_c = 2\pi f_c$

- Detection (or demodulation) of the sign
  - Envelope detection
    - Rectification and lowpass filtering
    - Noise at various frequencies cannot be rejected
    - Tuned amplifier or bandpass filter can remove some noise.
  - Phase-sensitive demodulation or synchronous detection
    - Multiplication or switching and lowpass filtering
    - Excellent noise rejection
- Phase-sensitive demodulation for $x(t) = \cos\omega_s t$
  - Assume the following,



$$
\begin{aligned}
v'(t) &= v_{AM}(t)\cos\omega_c t \\
&= x(t)\cos(\omega_c t + \theta)\cos\omega_c t \\
&= x(t)\frac{1}{2}\left\{\cos(2\omega_c t + \theta) + \cos\theta\right\}
\end{aligned}
$$

  - $y(t) = \frac{1}{2}\cos\theta\, x(t)$
    - If $\theta \neq \frac{\pi}{2}(2n+1)$ with $n = 0, 1, 2, \ldots$ and θ is constant, then we can detect $x(t)$ from $v_{AM}(t) = x(t)\cos\omega_c t$.
    - Noise not synchronized with $v_c(t) = \cos\omega_c t$ is rejected.

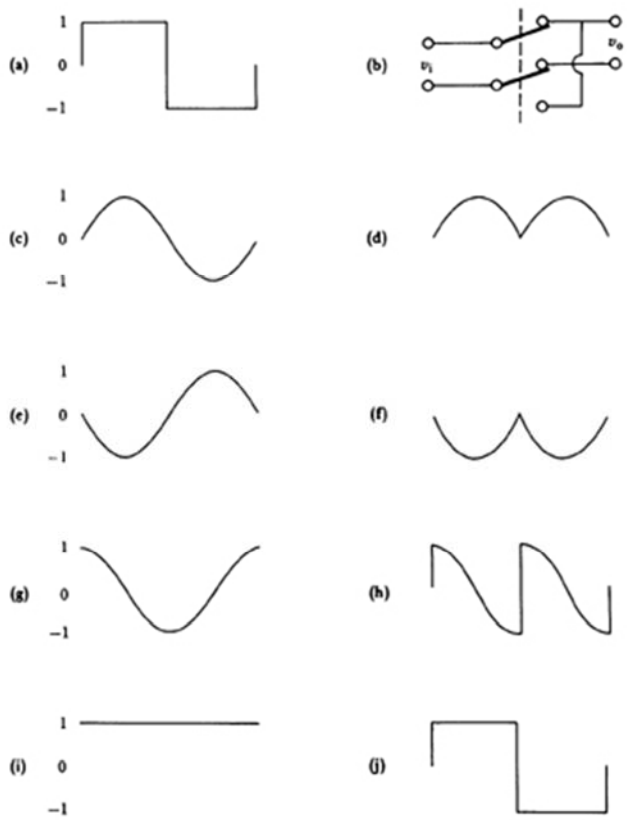


Figure 3.16 Functional operation of a phase-sensitive demodulator (a) Switching function. (b) Switch. (c), (e), (g), (i) Several input voltages. (d), (f), (h), (j) Corresponding output voltages.

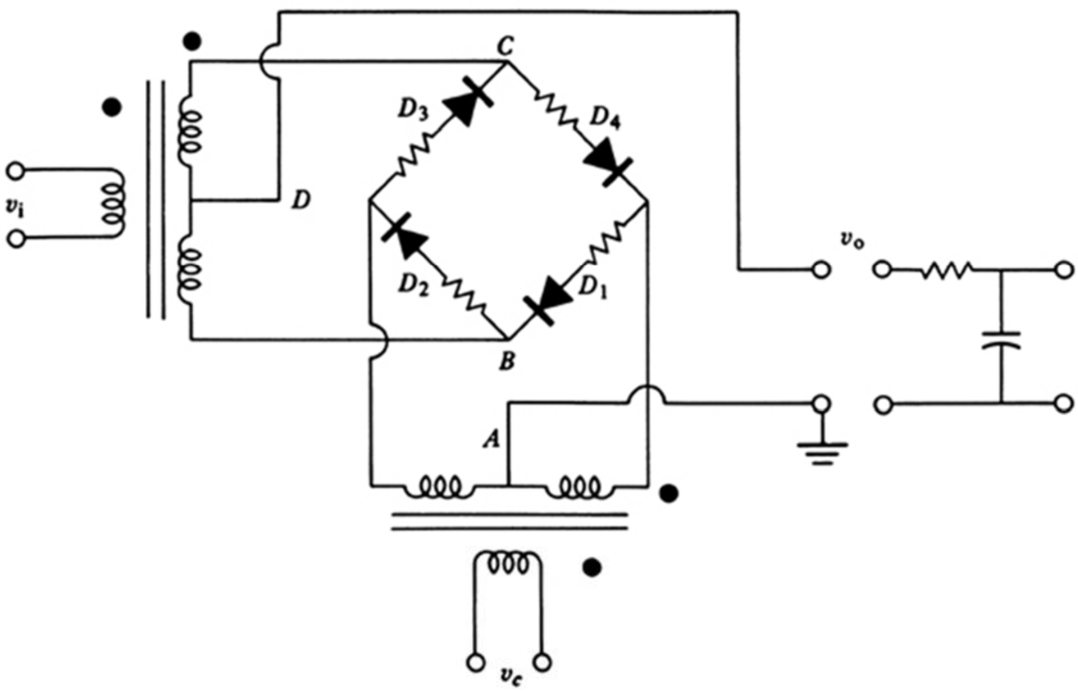
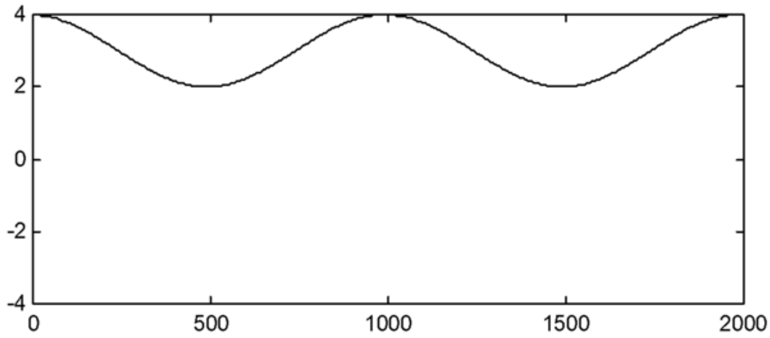
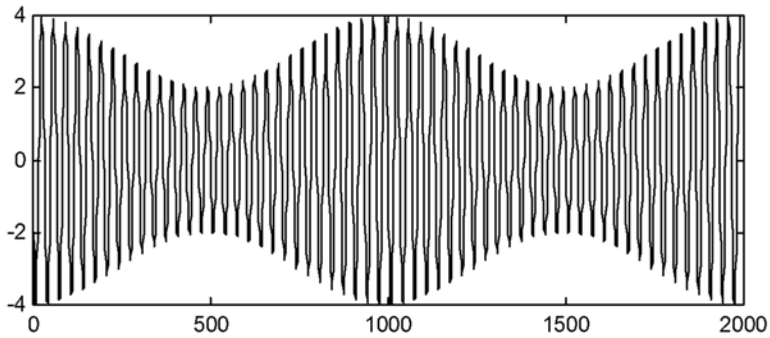


Figure 3.17 A ring demodulator This phase-sensitive detector produces a full-wave-rectified output υ_o that is positive when the input voltage υ_i is in phase with the carrier voltage υ_c and negative when υ_i is 180° out of phase with υ_c .

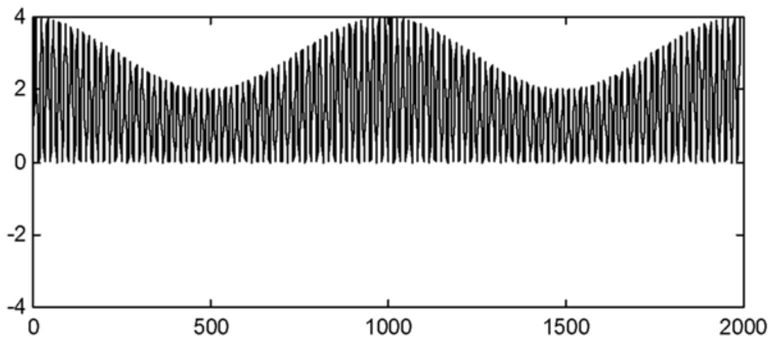
- Circuits: analog multiplier, balanced modulator/demodulator, ring demodulator



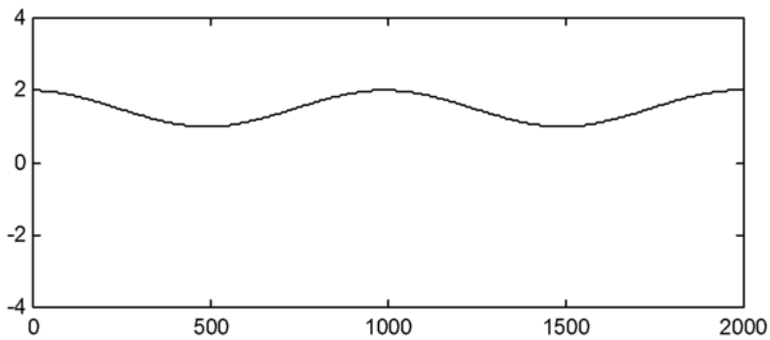
(a)



(b)



(c)



(d)

❍ Example

- (a) $x(t)=\cos(2\pi t)+3$
- (b) $v_{AM}(t)=\{\cos(2\pi t)+3\}\cos(2\pi\times 30t)$
- (c) $v'(t)=\{\cos(2\pi t)+3\}\cos^2(2\pi\times 30t)$
- (e) $y(t)=\frac{1}{2}\{\cos(2\pi t)+3\}$

3.16 Microcomputers in Medical Instrumentation