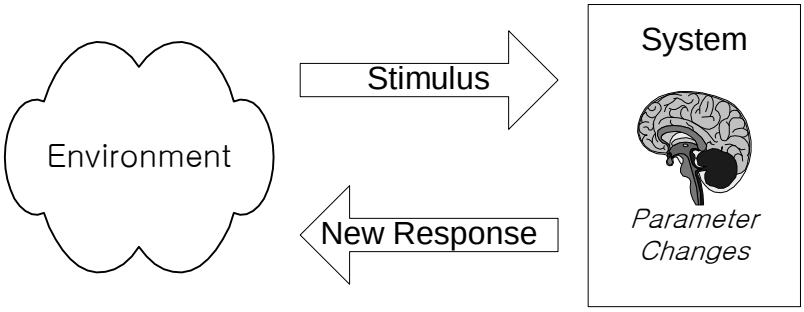
Learning Processes

❍ What is learning?



❍ Learning paradigm

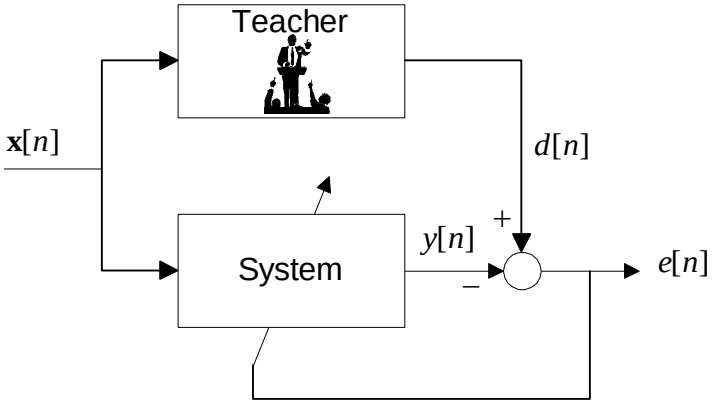
- Supervised learning: learning with a teacher
- Unsupervised learning or self-organized learning: learning without a teacher

❍ Learning algorithm

- Error-correction learning
- Memory-based learning
- Hebbian learning
- Competitive learning
- Stochastic learning

(1) Error-correction learning

• Supervised learning



• Delta rule or Widrow-Hoff rule: *LMS (least mean square) algorithm*

▫ Let the system parameter be $\mathbf{w}[n]$ and $y[n] = \mathbf{w}^T[n]\mathbf{x}[n]$. Then,

$$\mathbf{w}^* = \arg\min_{\mathbf{w}} \mathcal{E}[n] = \arg\min_{\mathbf{w}} \frac{1}{2} e^2[n].$$

▫ From LMS algorithm, $\mathbf{w}[n+1] = \mathbf{w}[n] + \eta e[n]\mathbf{x}[n]$.

▫ Step size parameter η is called as the "*learning rate parameter*"

• Batch mode algorithm: *LS (least square) algorithm*

(2) Memory-based learning

• Supervised learning

• Past experiences are explicitly stored in a large memory as $\{(\mathbf{x}_i, d_i)\}_{i=1}^{N}$

• Given a new input $\mathbf{x}_{\text{test}}$,

▫ Define a local neighborhood of $\mathbf{x}_{\text{test}}$

▫ Classification rule applied to the local neighborhood

- Nearest neighbor rule
- k-nearest neighbor classifier

(3) Hebbian learning

• Unsupervised learning

• Hebbian synapse

▫ If two neurons on either side of a synapse (connection) are activated simultaneously (or synchronously), then the strength of that synapse is selectively strengthened.

▫ If two neurons on either side of a synapse (connection) are activated asynchronously, then that synapse is selectively weakened or eliminated.

• Properties of Hebbian synapse

▫ *Time-dependent* mechanism

▫ *Local* mechanism

▫ *Interactive* mechanism

▫ Conjunctional or *corelational* mechanism

• Mathematical models: $\Delta w_{kj}[n] = F(y_k[n], x_j[n])$

▫ Hebb's hypothesis: $\Delta w_{kj}[n] = \eta y_k[n] x_j[n]$, exponential growth, may be saturated

▫ Covariance hypothesis: $\Delta w_{kj}[n] = \eta\left(y_k[n] - \overline{y_k}\right)\left(x_j[n] - \overline{x_j}\right)$ with time averages $\overline{y_k}$ and $\overline{x_j}$

▫ Modification using a forgetting factor:

$$\Delta w_{kj}[n] = \eta y_k[n] x_j[n] - \alpha y_k[n] w_{kj}[n] = \alpha y_k[n]\left(c x_j[n] - w_{kj}[n]\right)$$

(4) Competitive learning

- Unsupervised learning
- Competitive learning rule
  - Competition among neurons with the same structure but with different weights
  - Strength (output) of each neuron has a certain limit
  - The winner(s) is one (i.e., competitive) or more neurons (i.e., cooperative) with the biggest strength and called winner-takes-all neuron
- Mathematical models: $\Delta w_{kj}[n] = \begin{cases} \eta\left(x_j[n] - w_{kj}[n]\right) & \text{if winner} \\ 0 & \text{otherwise} \end{cases}$
- Clustering

Clustering and Classification

(1) Clustering

- Unsupervised learning
  - Labeling could be too cost
  - Understand internal structure of data distribution from clusters
  - Preprocessing for classification since features within the same cluster are similar

- *Clustering problem definition*. Given a set of vectors $\{\mathbf{x}_k\}_{k=1}^{K}$, find a set of C clustering centers $\{\mathbf{w}_i\}_{i=1}^{C}$ such that each $\mathbf{x}_k$ is assigned to a cluster $\mathbf{w}_i$ so that the average distortion

$$D = \frac{1}{K}\sum_{i=1}^{C}\sum_{k=1}^{K} I(\mathbf{x}_k, i) d(\mathbf{x}_k, \mathbf{w}_i)$$

where $d(\mathbf{x}_k, \mathbf{w}_i)$ is a distance measure and the indicator function is

$$I(\mathbf{x}_k, i) = \begin{cases} 1 & \text{if } d(\mathbf{x}_k, \mathbf{w}_i) < d(\mathbf{x}_k, \mathbf{w}_j), \forall j \neq i \\ 0 & \text{otherwise} \end{cases}.$$

- *K-means clustering algorithm*.
  - Initialization.

    Randomize $\{\mathbf{w}_i\}_{i=1}^{C}$, $I(\mathbf{x}_k, i)$=0 for $1 \le k \le K$ and $1 \le i \le C$, $D(0) = 0$, $n = 1$

  - Repeat.

    Compute $d(\mathbf{x}_k, \mathbf{w}_i)$ for $1 \le k \le K$ and $1 \le i \le C$

    Evaluate $I(\mathbf{x}_k, i) = \begin{cases} 1 & \text{if } d(\mathbf{x}_k, \mathbf{w}_i) < d(\mathbf{x}_k, \mathbf{w}_j), \forall j \neq i \\ 0 & \text{otherwise} \end{cases}$ for $1 \le k \le K$

    Compute $D[n] = \frac{1}{K}\sum_{i=1}^{C}\sum_{k=1}^{K} I(\mathbf{x}_k, i) d(\mathbf{x}_k, \mathbf{w}_i)$

    Update $\mathbf{w}_i = \frac{1}{N_i}\sum_{k=1}^{K} I(\mathbf{x}_k, i)\mathbf{x}_k$ with $N_i = \sum_{k=1}^{K} I(\mathbf{x}_k, i)$ for $1 \le i \le C$

    If $1 - \frac{D[n-1]}{D[n]} < \varepsilon$, stop

- *Distance measure*.

- Norm, $d(\mathbf{x},\mathbf{y}) = \|\mathbf{x}-\mathbf{y}\|_p$
- Mahalanobis distance, $d(\mathbf{x},\mathbf{y}) = (\mathbf{x}-\mathbf{y})^T \mathrm{S}_{xy}^{-1}(\mathbf{x}-\mathbf{y})$
- Angle, $d(\mathbf{x},\mathbf{y}) = \dfrac{\mathbf{x}^T\mathbf{y}}{\|\mathbf{x}\|_2 \, \|\mathbf{y}\|_2}$
- Tanimoto coefficient, $d(\mathbf{x},\mathbf{y}) = \dfrac{\mathbf{x}^T\mathbf{y}}{\mathbf{x}^T\mathbf{x}+\mathbf{y}^T\mathbf{y}-\mathbf{x}^T\mathbf{y}}$

• *Distortion measure*.

- Mean square error, $D = \dfrac{1}{K}\sum_{i=1}^{C}\sum_{k=1}^{K} I(\mathbf{x}_k, i)\|\mathbf{x}_k - \mathbf{w}_i\|_2^2 = \sum_{i=1}^{C}\dfrac{1}{N_i}\sum_{\mathbf{x},\mathbf{y}\in C(i)}\|\mathbf{x}-\mathbf{y}\|_2^2$
- In general, $D = \sum_{i=1}^{C}\dfrac{1}{N_i}\sum_{\mathbf{x},\mathbf{y}\in C(i)} d(\mathbf{x},\mathbf{y})$ or $D = \sum_{i=1}^{C}\left\{\min_{\mathbf{x},\mathbf{y}\in C(i)} d(\mathbf{x},\mathbf{y})\right\}$

• *Scattering criteria*.

- Mean of cluster i, $\mathbf{m}_i = \dfrac{1}{N_i}\sum_{k=1}^{K} I(\mathbf{x}_k, i)\mathbf{x}_k$
- Total mean, $\mathbf{m}_i = \dfrac{1}{K}\sum_{i=1}^{C} N_i\mathbf{m}_i$
- Scatter matrix of cluster i, $\mathbf{S}_i = \sum_{k=1}^{K} I(\mathbf{x}_k, i)\,(\mathbf{x}_k - \mathbf{m}_i)(\mathbf{x}_k - \mathbf{m}_i)^T$
- Within cluster scatter matrix, $\mathbf{S}_W = \sum_{i=1}^{C}\mathbf{S}_i$
- Between clusters scatter matrix, $\mathbf{S}_B = \sum_{i=1}^{C} N_i(\mathbf{m}_i - \mathbf{m})(\mathbf{m}_i - \mathbf{m})^T$
- Total scatter matrix, $\mathbf{S}_T = \mathbf{S}_W + \mathbf{S}_B = \sum_{k=1}^{K}(\mathbf{x}_k - \mathbf{m})(\mathbf{x}_k - \mathbf{m})^T$
- Note that $D = \mathrm{tr}\left(\mathbf{S}_B^{-1}\mathbf{S}_W\right)$
- Distance between clusters,

$$d_{\min}\{C(i),C(j)\} = \min_{\mathbf{x}\in C(i),\mathbf{y}\in C(j)} d(\mathbf{x},\mathbf{y})$$

$$d_{\max}\{C(i),C(j)\} = \max_{\mathbf{x}\in C(i),\mathbf{y}\in C(j)} d(\mathbf{x},\mathbf{y})$$

$$d_{avg}\{C(i),C(j)\} = \frac{1}{N_i N_j}\sum_{\mathbf{x}\in C(i)}\sum_{\mathbf{y}\in C(j)} d(\mathbf{x},\mathbf{y})$$

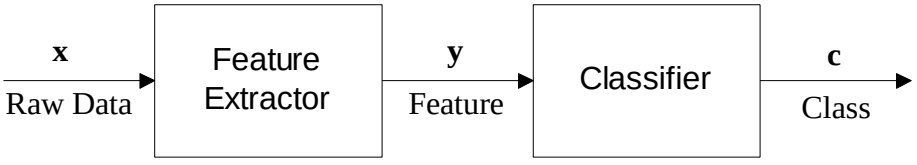
$$d_{mean}\{C(i),C(j)\} = \left|\mathbf{m}_i - \mathbf{m}_j\right|$$

- *Hierarchical clustering*.
  - Merge. Initially, each $\mathbf{x}_k$ is a cluster. During iterations, nearest pair of distinct clusters are merged.
  - Split. Initially, all $\{\mathbf{x}_k\}_{k=1}^{K}$ belong to one cluster. During iterations, one cluster is spitted into two or more clusters if within cluster scattering is large.

(2) Classification

- *Classification problem definition*. Assume data samples $X = \{\mathbf{x}_k\}_{k=1}^{K}$ are drawn from M classes $C = \{C(i)\}_{i=1}^{M}$. Given an observation $\mathbf{x}$, find a decision rule $g(\mathbf{x}) \in C$ such that the probability of classification $\sum_{i=1}^{M} \Pr\{g(\mathbf{x}) = C(i) | \mathbf{x} \in C(i)\}$ is maximized.
- *Nearest neighbor classifier*. Assume that the already classified set of data or mapping or ANN is available, i.e., we have $\{(\mathbf{y}_i, C(j))\}$ for $1 \le i \le N$ and $1 \le j \le M$. For a new sample $\mathbf{x}$, the decision rule choose $g(\mathbf{x}) = C(k)$ if $\mathbf{y}^* = \arg\min_{\mathbf{y}_i} \|\mathbf{y}_i - \mathbf{x}\|$ is paired with $C(k)$.
- *k-nearest neighbor classifier*. Examine k nearest classes and classify $\mathbf{x}$ into the majority of them.
- *Statistical decision rules*.
  - Maximum posterior probability (MAP) classifier
  - Maximum Likelihood (ML) classifier
  - Neyman-Pearson (NP) detector
  - Bayes detector

(3) Features



- *Feature representation*.

- Symbolic vs. numeric.
- Higher dimensional features.

• *Feature selection*.

- Select a subset of available features can improve classification.
- Selection of subspace or subspace approximation.
- Hidden neurons in MPL are feature detectors. Hidden neuron pruning is a kind of feature selection.

• *Feature transformation*.

- Affine transformation. $y = Tx + b$
- Rotation
- Linear filtering
- Fourier transform (DFT)
- Discrete cosine transform
- Karhunen-Loeve expansion (principal component analysis)
- Eigendecomposition
- Edge or line detection
- Other linear or nonlinear operation

(4) Data sampling

• Sample data independently from the underlying population.

• Use resampling with randomization.

• Use *M*-fold cross-validation or leave-one-out cross-validation.

Artificial Neural Network (ANN)

❍ An (artificial) neural network is a massively parallel distributed processor made up of simple processing units, which has a natural propensity for storing experimental knowledge and making it available for use. It resembles the brain in two respects:

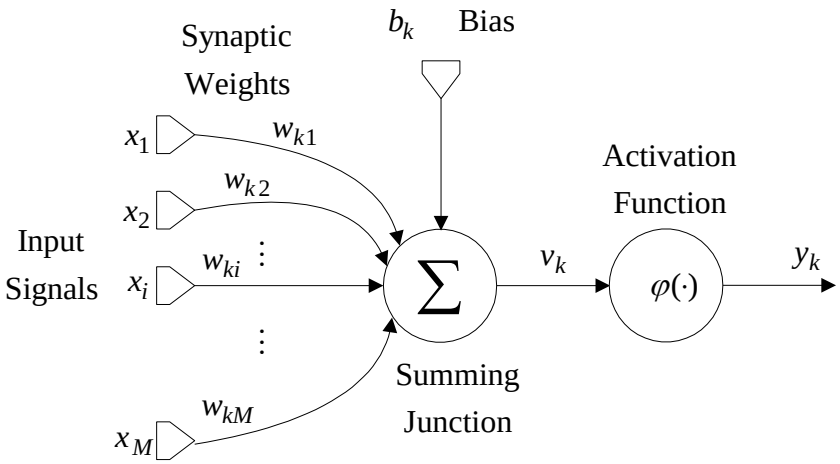
● Knowledge is acquired by the network from its environment through a learning process.

● Interneuron connection strengths, known as synaptic weights, are used to store the acquired knowledge.

❍ Properties of artificial neural networks

- ● Nonlinearity
- ● Input-output mapping
- ● Adaptivity
- ● Evidential response
- ● Contextual information
- ● Fault tolerance
- ● VLSI implementability
- ● Uniformity of analysis and design
- ● Neurobiological analogy

(1) Models of a neuron



• *Neuron* is an information processing unit.

- ▫ A set of synapses or connecting links with a weight or strength
- ▫ Adder or linear combiner
- ▫ Activation function or squashing function

$$v_k = \sum_{j=0}^{M} w_{kj} x_j \quad \text{with} \quad w_{k0} = b_k, \quad x_0 = 1 \quad \text{and} \quad y_k = \varphi(v_k)$$

• *Activation function*.

▫ *Threshold function* or Heaviside function ⇒ McCulloch-Pitts model, all-or-none

$$y = \varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases} \quad \text{or}$$

the signum function

$$y = \varphi(v) = \operatorname{sgn}(v) = \begin{cases} 1 & \text{if } v > 0 \\ 0 & \text{if } v = 0 \\ -1 & \text{if } v < 0 \end{cases}$$

▫ *Piecewise linear function* (can have a gain)

$$y = \varphi(v) = \begin{cases} 1 & \text{if } v \geq +\frac{1}{2} \\ v_k & \text{if } -\frac{1}{2} < v < +\frac{1}{2} \\ 0 & \text{if } v \leq -\frac{1}{2} \end{cases} \quad \text{or} \quad y = \varphi(v) = \begin{cases} 1 & \text{if } v \geq +1 \\ v_k & \text{if } -1 < v < +1 \\ -1 & \text{if } v \leq -1 \end{cases}$$

▫ *Sigmoid function*: strictly increasing function with a graceful balance between linear and nonlinear behavior, for example *logistic function*

$$y = \varphi(v) = \frac{1}{1 + \exp(-av)} \quad \text{or}$$

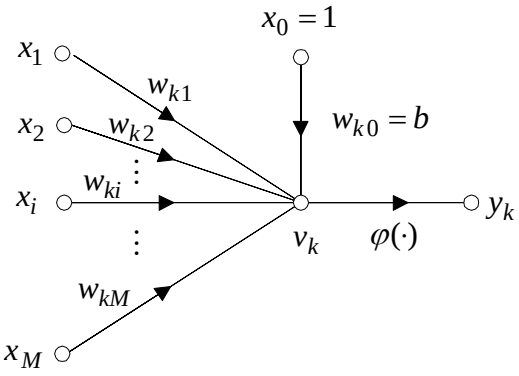
the *hyperbolic tangent function*

$$y = \varphi(v) = \tanh(v)$$

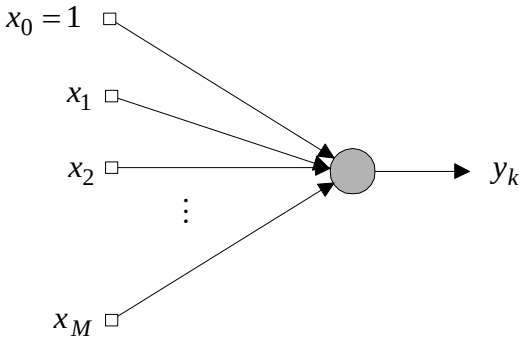
• Stochastic model.

(2) Signal flow graph, architectural graph, and Matlab representation

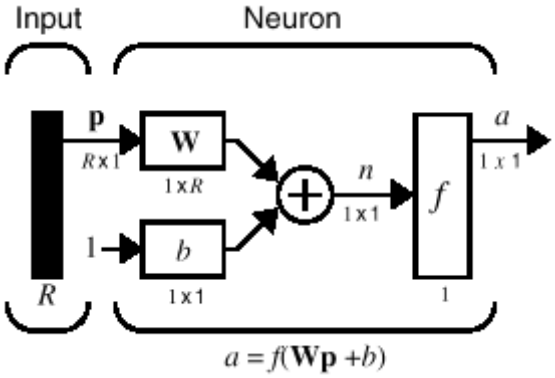
• *Signal flow graph*.



• *Architectural graph*.



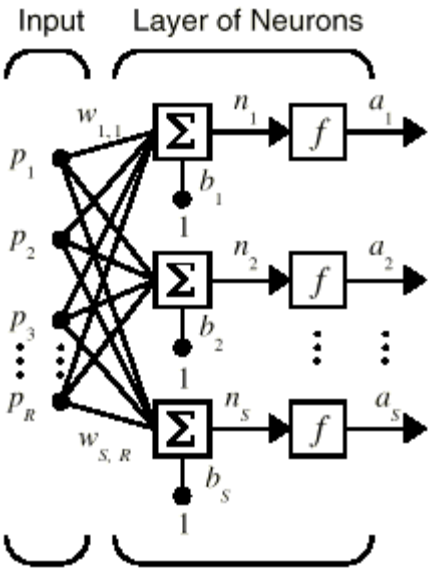
• *Matlab representation*.

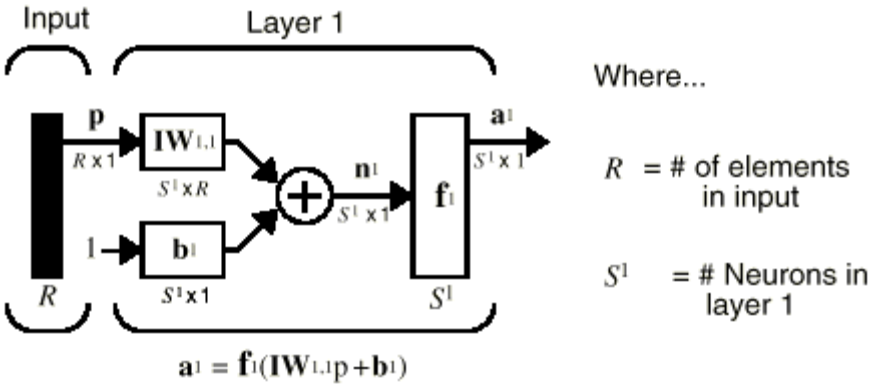


($R = M$)

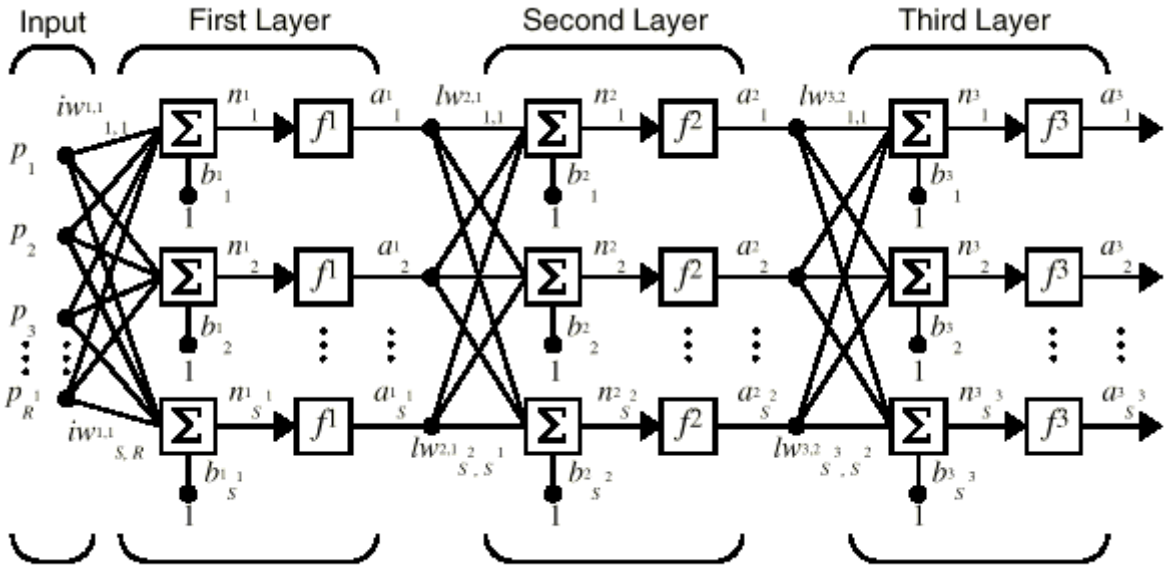
(3) Network architecture

• *Single-layer feedforward network*.

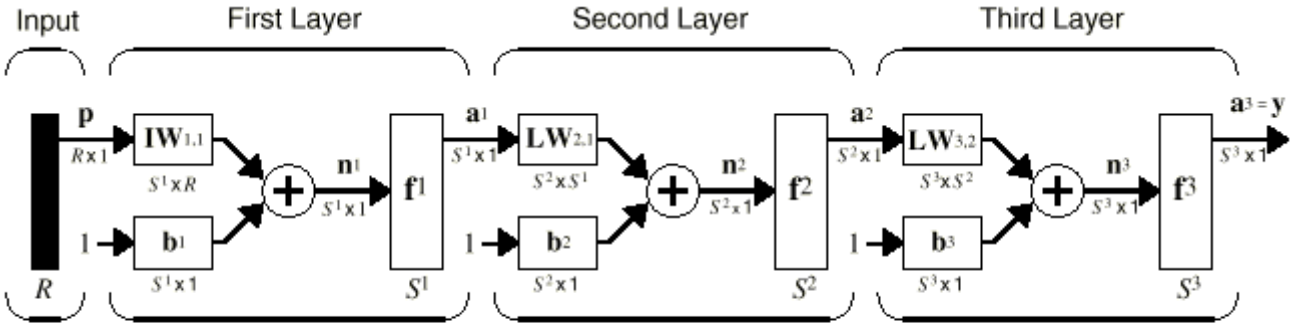




• *Multilayer feedforward network*.



$\mathbf{a}^1 = \mathbf{f}^1(\mathbf{IW}^{1,1}\mathbf{p}+\mathbf{b}^1)$ $\mathbf{a}^2 = \mathbf{f}^2(\mathbf{LW}^{2,1}\mathbf{a}^1+\mathbf{b}^2)$ $\mathbf{a}^3 = \mathbf{f}^3(\mathbf{LW}^{3,2}\mathbf{a}^2+\mathbf{b}^3)$



$\mathbf{a}^1 = \mathbf{f}^1(\mathbf{IW}^{1,1}\mathbf{p}+\mathbf{b}^1)$ $\mathbf{a}^2 = \mathbf{f}^2(\mathbf{LW}^{2,1}\mathbf{a}^1+\mathbf{b}^2)$ $\mathbf{a}^3 = \mathbf{f}^3(\mathbf{LW}^{3,2}\mathbf{a}^2+\mathbf{b}^3)$

$$\mathbf{a}^3 = \mathbf{f}^3(\mathbf{LW}^{3,2}\mathbf{f}^2(\mathbf{LW}^{2,1}\mathbf{f}^1(\mathbf{IW}^{1,1}\mathbf{p}+\mathbf{b}^1)+\mathbf{b}^2)+\mathbf{b}^3 = \mathbf{y}$$

- *Recurrent network* or *dynamic network*.

(4) Knowledge representation

- *Knowledge* refers to stored information or model used by a person or machine to interpret, predict, and appropriately respond to the outside world.
- *Information*.
  - *Prior information*
  - *Observations* or *measurements* provide a pool of information from which the *examples* are drawn to train the ANN
- *Examples* ⇒ a set of training data or training samples
  - *Labeled* ⇒ supervised learning

▫ *Unlabelled* ⇒ unsupervised learning

• *Four rules of knowledge representation for ANN*.

▫ *Rule 1*. Similar inputs from similar class should usually produce similar representation inside the network, and should therefore be classified as belonging to the same category.

▫ *Rule 2*. Items to be categorized as separate classes should be given widely different representations in the network.

▫ *Rule 3*. If a particular feature is important, then there should be a large number of neurons involved in the representation of that item in the network.

▫ *Rule 4*. Prior information and variances should be built into the design of a neural network, thereby simplifying the network design by not having to learn them.

⇒ "In general, use your common sense."

• *Training and generalization*.

Single-Layer Perceptron

BACKGROUND MATERIALS

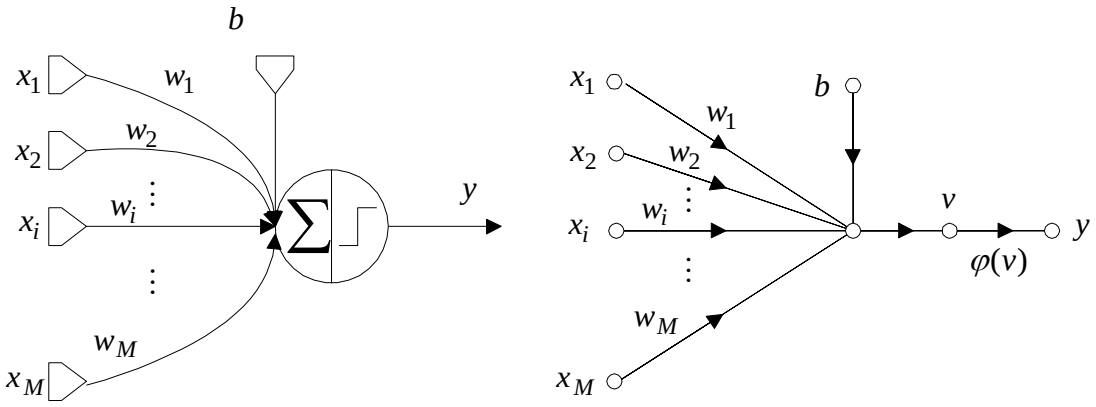
❍ Unconstrained optimization techniques
- ● Steepest descent
- ● Newton's method
- ● Gauss-Newton method

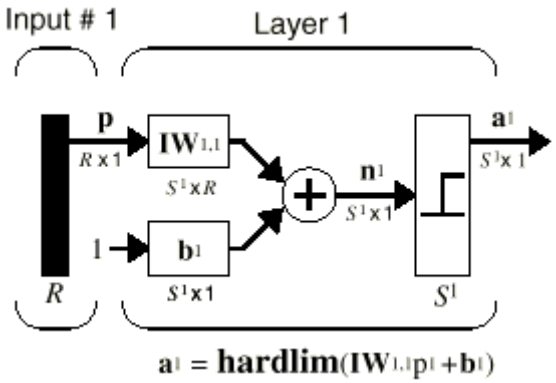
❍ Wiener filter

❍ Adaptive filter using LMS (least mean square) algorithm

❍ LS (least square) method

(1) Perceptron





• Decision boundary is a *hyperplane*, $v = \sum_{i=1}^{M} w_i x_i + b = \mathbf{w}^T\mathbf{x} + b = 0$. And

$$y = \begin{cases} 1 \Leftrightarrow \mathbf{x} \text{ belong to class } C_1 \Leftrightarrow \mathbf{w}^T\mathbf{x} > 0 \\ 0 \Leftrightarrow \mathbf{x} \text{ belong to class } C_2 \Leftrightarrow \mathbf{w}^T\mathbf{x} \le 0 \end{cases}.$$

• *Perceptron convergence algorithm*

▫ Let $\mathbf{x}[n] = \left[+1, x_1[n], \cdots, x_M[n]\right]^T$ and $\mathbf{w}[n] = \left[b[n], w_1[n], \cdots, w_M[n]\right]^T$.

▫ *Initialization*. $n = 0$ and $\mathbf{w}[0] = \mathbf{0}$.

▫ *Activation*. Apply $\mathbf{x}[n]$ and get $d[n]$.

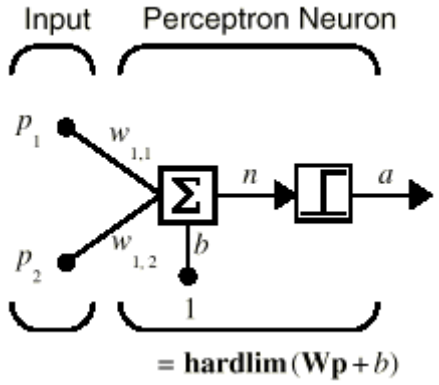
▫ *Response*. $y[n] = \text{sgn}\left(\mathbf{w}^T[n]\mathbf{x}[n]\right)$

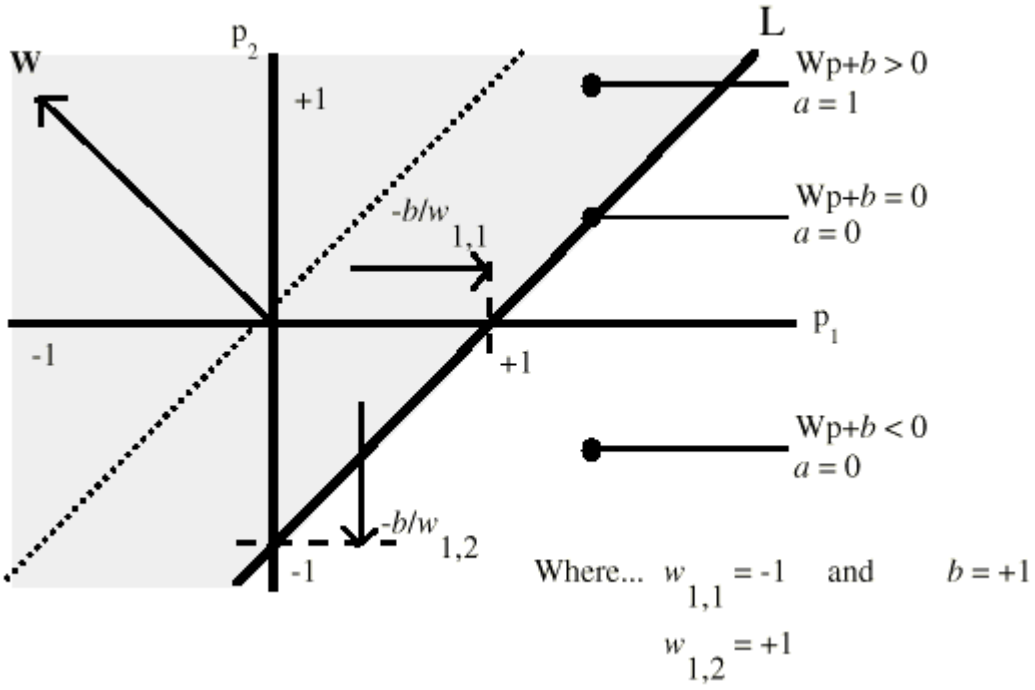
▫ *Weight adaptation (LMS)*. $\mathbf{w}[n+1] = \mathbf{w}[n] + \eta\left(d[n] - y[n]\right)\mathbf{x}[n]$ where

$$d[n] = \begin{cases} 1, & \text{if } \mathbf{x} \text{ belong to class } C_1 \\ -1, & \text{if } \mathbf{x} \text{ belong to class } C_2 \end{cases}$$

(2) Perceptron as a linear classifier (Matlab)

• newp
• sim
• init
• learnp
• adapt





(3) Limitations of perceptron

• XOR problem

Multilayer Perceptron

❍ Multilayer perceptron (MLP)
- ● Input layer
- ● Hidden layer
- ● Output layer

❍ Feed forward

❍ Nonlinear activation function

❍ Backpropagation learning algorithm

(1) Structure of MLP

(2) Backpropagation learning algorithm

• Epoch: one complete presentation of the complete training samples

• *At the output layer*, at iteration *n* (i.e., *n*th training example)

- ▫ $e_j[n] = d_j[n] - y_j[n]$ at *j*th neuron
- ▫ $\mathcal{E}[n] = \frac{1}{2}\sum_{j\in C} e_j^2[n]$ where *C* is a set of all neurons at the output layer
- ▫ $v_j[n] = \sum_{i=0}^{M} w_{ji} y_i[n]$ and $y_j[n] = \varphi_j\left(v_j[n]\right)$
- ▫ *Chain rule*.

$$\frac{\partial \mathcal{E}[n]}{\partial w_{ji}[n]} = \frac{\partial \mathcal{E}[n]}{\partial e_j[n]}\frac{\partial e_j[n]}{\partial y_j[n]}\frac{\partial y_j[n]}{\partial v_j[n]}\frac{\partial v_j[n]}{\partial w_{ji}[n]}$$
$$= e_j[n](-1)\varphi_j^{\mathrm{n}}\left(v_j[n]\right) y_i[n]$$

- ▫ *LMS algorithm*. $\Delta w_{ji} = -\eta\frac{\partial \mathcal{E}[n]}{\partial w_{ji}[n]} = \eta e_j[n]\varphi_j^{\mathrm{n}}\left(v_j[n]\right) y_i[n] = \eta\delta_j[n] y_i[n]$ with

$$\delta_j[n] = -\frac{\partial \mathcal{E}[n]}{\partial v_j[n]} = -\frac{\partial \mathcal{E}[n]}{\partial e_j[n]}\frac{\partial e_j[n]}{\partial y_j[n]}\frac{\partial y_j[n]}{\partial v_j[n]} = e_j[n]\varphi_j^{\mathrm{n}}\left(v_j[n]\right)$$

• *At a hidden layer*, at iteration *n* (i.e., *n*th training example)

- ▫ At *j*th neuron, $\delta_j[n] = -\frac{\partial \mathcal{E}[n]}{\partial v_j[n]} = -\frac{\partial \mathcal{E}[n]}{\partial y_j[n]}\frac{\partial y_j[n]}{\partial v_j[n]} = -\frac{\partial \mathcal{E}[n]}{\partial y_j[n]}\varphi_j^{\mathrm{n}}\left(v_j[n]\right)$
- ▫ From $\mathcal{E}[n] = \frac{1}{2}\sum_{k\in C} e_k^2[n]$, $\frac{\partial \mathcal{E}[n]}{\partial y_j[n]} = \sum_k e_k[n]\frac{\partial e_k[n]}{\partial y_j[n]} = \sum_k e_k[n]\frac{\partial e_k[n]}{\partial v_k[n]}\frac{\partial v_k[n]}{\partial y_j[n]}$

▫ Since $e_k[n] = d_k[n] - y_k[n] = d_k[n] - \varphi_k\left(v_k[n]\right)$ for the output layer neuron *k*,

$$\frac{\partial e_k[n]}{\partial v_k[n]} = -\varphi_k^{\mathrm{n}}\left(v_k[n]\right).$$

▫ Since $v_k[n] = \sum_{j=0}^{M} w_{kj} y_j[n]$, $\frac{\partial v_k[n]}{\partial y_j[n]} = w_{kj}[n]$.

▫ Therefore, $\frac{\partial \mathcal{E}[n]}{\partial y_j[n]} = -\sum_k e_k[n]\varphi_k^{\mathrm{n}}\left(v_k[n]\right) w_{kj}[n] = -\sum_k \delta_k[n] w_{kj}[n]$.

▫ Finally, at *j*th neuron of the hidden layer, $\delta_j[n] = \varphi_j^{\mathrm{n}}\left(v_j[n]\right)\sum_k \delta_k[n] w_{kj}[n]$.

▫ *LMS algorithm*. $\Delta w_{ji} = \eta \delta_j[n] y_i[n]$.

• *Activation functions*

▫ Logistic function.

$$y_j[n] = \varphi_j\left(v_j[n]\right) = \frac{1}{1+\exp\left(-a v_j[n]\right)},\ a > 0 \text{ and } -\infty < v_j[n] < \infty,$$

$$\varphi_j^{\mathrm{n}}\left(v_j[n]\right) = a y_j[n]\left(1 - y_j[n]\right), \text{ and}$$

$$\begin{aligned}\delta_j[n] &= e_j[n]\varphi_j^{\mathrm{n}}\left(v_j[n]\right) \\ &= \begin{cases} a\left(d_j[n] - y_j[n]\right) y_j[n]\left(1 - y_j[n]\right) & \text{for output layer} \\ a y_j[n]\left(1 - y_j[n]\right)\sum_k \delta_k[n] w_{kj}[n] & \text{for hidden layer} \end{cases}.\end{aligned}$$

▫ Hyperbolic tangent function.

$$y_j[n] = \varphi_j\left(v_j[n]\right) = a\tanh\left(b v_j[n]\right),\ a,b > 0 \text{ and } -\infty < v_j[n] < \infty,$$

$$\varphi_j^{\mathrm{n}}\left(v_j[n]\right) = \frac{b}{a}\left(a - y_j[n]\right)\left(a + y_j[n]\right), \text{ and}$$

$$\begin{aligned}\delta_j[n] &= e_j[n]\varphi_j^{\mathrm{n}}\left(v_j[n]\right) \\ &= \begin{cases} \frac{b}{a}\left(d_j[n] - y_j[n]\right)\left(a - y_j[n]\right)\left(a + y_j[n]\right) & \text{for output layer} \\ \frac{b}{a}\left(a - y_j[n]\right)\left(a + y_j[n]\right)\sum_k \delta_k[n] w_{kj}[n] & \text{for hidden layer} \end{cases}.\end{aligned}$$

• *Momentum*. $\Delta w_{ji}[n] = \alpha \Delta w_{ji}[n-1] + \eta \delta_j[n] y_i[n]$ $\Rightarrow$ stabilizing effect

• *Modes of training*.

▫ A set of training examples, $\{(\mathbf{x}[i], d[i])\}_{i=1}^{N}$ $\Rightarrow$ epoch

▫ Randomize samples at each epoch

▫ Sequential mode (on-line, pattern, or stochastic mode): update weight sample by sample

▫ Batch mode (on-line, pattern, or stochastic mode): update weight at the end of epoch

$$\mathcal{E}_{av} = \frac{1}{N}\sum_{n=1}^{N}\mathcal{E}[n] \quad \text{and} \quad \Delta w_{ji} = -\eta\frac{\partial \mathcal{E}_{av}}{\partial w_{ji}} = -\frac{\eta}{N}\sum_{n=1}^{N} e_j[n]\frac{\partial e_j[n]}{\partial w_{ji}}$$

• *Stopping criterion*.

▫ Small norm of the gradient vector

▫ Small absolute value of change in the average squared error per epoch

(3) Heuristics

• When the training data set is large and redundant, sequential mode is usually faster and better.

• When the training data set is not large, there are several batch mode algorithms that are faster.

• Information content of a training example

▫ Use an example that results in the largest training error

▫ Use an example that is different from all those previously used

▫ Distribution of training examples should not be distorted

▫ Avoid any outlier in the training data set

• Activation function

• Target value

• Input normalization

• Initialization

• Use any prior information

• Learning rate and momentum

▫ Every adjustable network parameter should have its own individual learning rate parameter.

▫ Every learning rate parameter should be allowed to vary from one iteration to the next.

▫ When the derivative of the cost function w.r.t. a synaptic weight has the same

algebraic sign for several consecutive iterations, the corresponding learning rate parameter should be increased.

▫ When the algebraic sign of the derivative of the cost function w.r.t. a synaptic weight alternates for several consecutive iterations, the corresponding learning rate parameter should be decreased.

(4) Output representation and decision rule

• ANN is already trained

• Consider *M*-class classification problem

▫ Let $\mathbf{x}_j$ denote *j*th sample (prototype) to be classified and ANN produces output

$$\mathbf{y}_j = \left[y_{1,j}, \cdots, y_{M,j} \right]^T = \left[F_1(\mathbf{x}_j), \cdots, F_M(\mathbf{x}_j) \right]^T = \mathbf{F}(\mathbf{x}_j).$$

▫ Note that the function **F** depends on the training data set $\left\{ \left(\mathbf{x}[i], d[i] \right) \right\}_{i=1}^{N}$.

▫ What is the optimal decision rule for classifying the *M* outputs of ANN?

▫ Assigning single class from *M* distinct classes:

$$\mathbf{x}_j \in C_k \text{ if } F_k(\mathbf{x}_j) > F_l(\mathbf{x}_j) \text{ for all } l \neq k$$

▫ Assigning multiple class from *M* distinct classes:

$$\mathbf{x}_j \in C_k \text{ if } F_k(\mathbf{x}_j) > \text{threshold } (\text{ex, } 0.5)$$

(5) Generalization

• A network is said to *generalize* well when the input-output mapping is correct for test data never used in creating or training the network.

• *Overtraining* or *overfitting* problem

• *Bias-variance trade-off*

• Factors influencing generalization

▫ Size of training set and how representative it is of the environment of interest

▫ Architecture of ANN

▫ Physical complexity of the problem at hand

(6) Cross-validation

• Backpropagation learning algorithm encodes an input-output mapping into the synaptic weights and thresholds of a MLP.

- For better generalization, partition the training set into two subsets.
  - Estimation subset, used to train or select the model
  - Validation subset, used to test or validate the model
- Early stopping rule, stop training when the error using validation subset starts increasing

(7) Network growing and pruning techniques

- Network growing
- Network pruning

(8) Supervised learning viewed as an optimization problem

- Conjugate gradient method
- Quasi-Newton method

(9) Matlab experiments

newff

init

sim

adapt: leardgd, leardgdm

train: traingd, traingdm, traingda, traingdx, trainrp, traincgf, traincgb, trainscg, trainbfg, trainoss, trainlm, trainbr

premnmx, postmnmx, tramnmx, prestd, poststd, trastd

prepca, prapca

postreg